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Labeled PSI from Homomorphic Encryption with Reduced Computation and Communication

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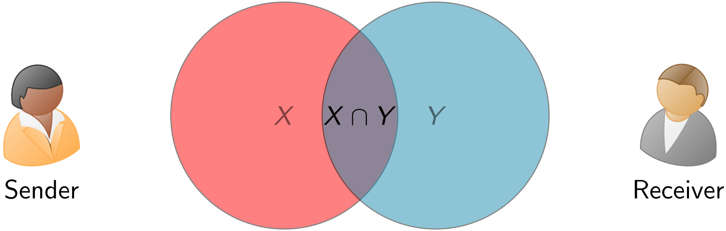
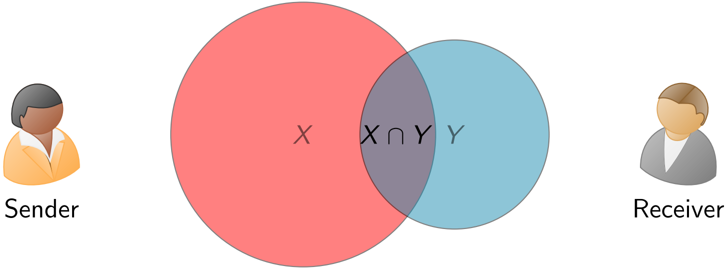
Private Set Intersection



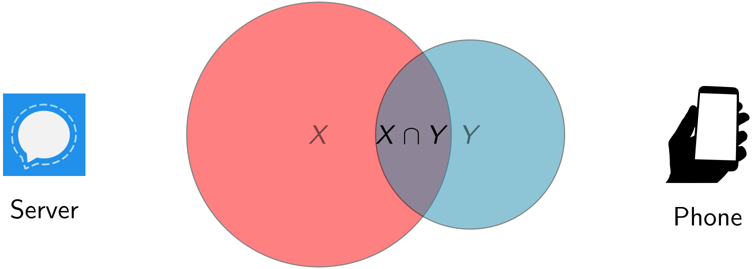
- Receiver learns $X \cap Y$.
- X and Y remain private.

Unbalanced PSI



- Unbalanced PSI - assume $|X| \gg |Y|$.

Private Contact Discovery Application



- X: registered phone numbers
- Y: contacts on the phone

Unbalanced PSI: Related Work

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Based on OPRF
Kales et al. USENIX'19

- Sender distributes cuckoo filter created from X
- Communication is $\mathcal{O}(|X|)$
- Very efficient online phase

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Based on HE
Chen et al. CCS'18

- Intersection is computed by the sender
- Communication is $\mathcal{O}(|Y| \log |X|)$
- Computation is $\mathcal{O}(|X|)$
- Starting point of our work

(Somewhat) Homomorphic Encryption

Functionality of HE

- $f(\mathrm{Ctxt}(Y)) = \mathrm{Ctxt}(f'(Y))$
- f' is any arithmetic circuit of bounded depth, e.g., $+, -, \cdot, \mathrm{Aut}$
- e.g., $f'(Y) = X \cap Y$, where X is hardwired

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Cost of HE

- Multiplication is the most expensive
- Need to minimize multiplicative width and depth
- Operations can be parallelized (more on this later)

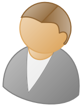
Basic PSI Protocol Using HE

Inputs: Sender inputs set X, receiver inputs set Y, $|X| \gg |Y|$

Setup: Receiver generates a key pair for the HE scheme.

Set encryption: $[\![y_1]\!], \ldots, [\![y_{|Y|}]\!]$

Compute intersection: $[\![d_1]\!], \ldots, [\![d_{|Y|}]\!]$



$$[\![d_i]\!] = r_i \prod_{x \in X} ([\![y_i]\!] - x)$$

Reply extraction: Receiver decrypts the ciphertexts and outputs

$$X \cap Y = \{y_i : \text{HE.Decrypt}([\![d_i]\!]) = 0\}$$

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Basic PSI Protocol Using HE

- Intersection polynomial

$$r \prod_{x \in X} (\llbracket y \rrbracket - x) = r\llbracket y \rrbracket^{|X|} + ra_{|X|}\llbracket y \rrbracket^{|X|-1} + \ldots + ra_0$$

- Multiplicative depth is $\mathcal{O}(\log |X|)$ from square and multiply
- Communication cost is $\mathcal{O}(|Y|)$ HE ciphertexts
- Computation cost is $\mathcal{O}(|X| \cdot |Y|)$ homomorphic operations

Previous Work
Windowing

- Instead of sending a single $[\![y]\!]$
- Send powers of $[\![y]\!]$, e.g., $[\![y^{2^0}]\!], [\![y^{2^1}]\!], \ldots, [\![y^{2^{\log|X|}}]\!]$
- New multiplicative depth $\mathcal{O}(\log\log|X|)$
- Communication increased by a factor of $\mathcal{O}(\log|X|)$

Previous Work

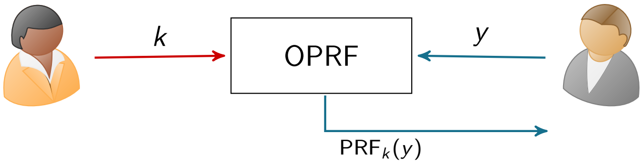
Parallel computation

	pt_0	pt_1	pt_2		ct_0
slot 3	$x_2^{(0)}$	$x_3^{(1)}$	$x_4^{(0)}$	$P_3(x) = (x - x_2^{(0)})(x - x_3^{(1)})(x - x_4^{(0)})$	y_1
slot 2	$x_0^{(1)}$	$x_3^{(0)}$		$P_2(x) = (x - x_0^{(1)})(x - x_3^{(0)})$	y_0
slot 1	$x_1^{(0)}$	$x_2^{(1)}$		$P_1(x) = (x - x_1^{(0)})(x - x_2^{(1)})$	y_2
slot 0	$x_0^{(0)}$	$x_1^{(1)}$	$x_4^{(1)}$	$P_0(x) = (x - x_0^{(0)})(x - x_1^{(1)})(x - x_4^{(1)})$	

- Use cuckoo hashing for Y
- Same for X but without eviction, hash x_i into $x_i^{(0)}$ and $x_i^{(1)}$
- Polynomials are evaluated in parallel!

Previous Work

OPRF preprocessing



- No need padding or randomizing the intersection polynomial
- Security against malicious receiver

Our Improvements

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General optimizations

- Fast OPRF from FourQ (Costello and Longa 2015).
- Polynomial evaluation with Paterson-Stockmeyer algorithm.

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Improved computation and communication

- Operations over prime fields.
- Extremal postage stamp bases.
- Implemented with SEAL.

Optimizing for communication complexity

- Operations over extension fields.
- Depth-free homomorphic Frobenius automorphisms.
- Implemented with HElib.

General optimizations

Paterson-Stockmeyer algorithm

Compute the degree D intersection polynomial in $\mathcal{O}(\sqrt{D})$ ciphertext-ciphertext multiplications.

The sender computes two sets of powers:

- **Low powers** $[\![y]\!]^2, [\![y]\!]^3, \dots, [\![y]\!]^{L-1}$
- **High powers:** $[\![y]\!]^L, [\![y]\!]^{2L}, [\![y]\!]^{3L}, \dots, [\![y]\!]^{(H-1)\cdot L}$

with $L, H \approx \sqrt{D}$.

General optimizations

Paterson-Stockmeyer algorithm

Then, rewrite the intersection polynomial:

$$\sum_{i=0}^{D} a_i \cdot [\![y]\!]^i$$

$$\Downarrow$$

$$\sum_{i=0}^{H-1} [\![y]\!]^{iL} \left(\sum_{j=0}^{L-1} \left(a_{iL+j} \cdot [\![y]\!]^j \right) \right)$$

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- **Non-scalar multiplicative complexity:** $\mathcal{O}(\sqrt{D})$

Improved computation and communication

Extremal postage stamp bases

How to minimize the number of powers sent without exceeding the target depth?

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Global postage-stamp problem

Given positive integers h and k, determine a set of k positive integers $A_k = \{a_1 = 1 < a_2 < \ldots < a_k\}$ such that all integers $1, 2, \ldots, n$ can be written as a sum of h or fewer of the a_j, and n is as large as possible.

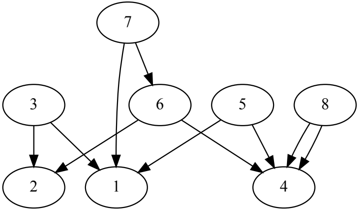
The set A_k is called an extremal postage-stamp basis.

Improved computation and communication

Extremal postage stamp bases

Computing powers of the query...

- when using windowing



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Improved computation and communication

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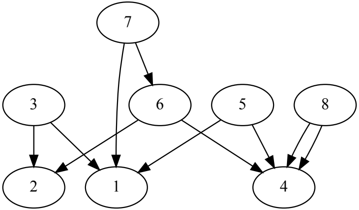
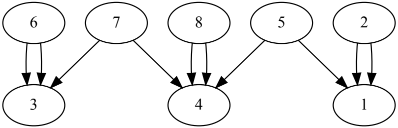
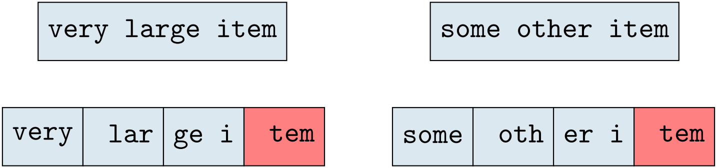


- when using extremal postage stamp bases



Improved computation and communication

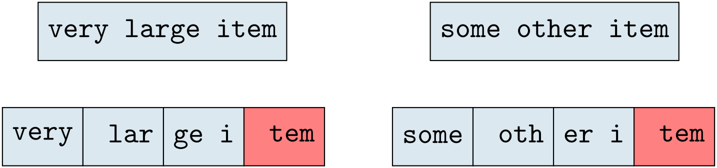
Dealing with large items



- Split items into multiple parts.

Improved computation and communication

Dealing with large items



- Split items into multiple parts.
- Perform OPRF before splitting the items to protect from partial item leakage.

Improved computation and communication

Results

$\lvert X \rvert$	$\lvert Y \rvert$	Protocol	Sender offline (s)	Sender online (s)
2^{28}	1024	This work (T=24)	3,680	7.80
		Chen et al. (T=32)	4,628	12.1
		LowMC-GC-PSI	1,869	0.93
		ECC-NR-PSI	52,332	1.34
2^{20}	5535	This work	28	3.23
		Chen et al.	43	4.23
		LowMC-GC-PSI	7.3	5.63
		ECC-NR-PSI	242	5.93

Improved computation and communication

Results

$\lvert X \rvert$	$\lvert Y \rvert$	Protocol	Offline comm. and receiver storage (MB)	Comm. (MB)
2^{28}	1024	This work (T=24)	0	6.08
		Chen et al. (T=32)	0	18.57
		LowMC-GC-PSI	1,072	24.01
		ECC-NR-PSI	1,072	6.06
2^{20}	5535	This work	0	5.39
		Chen et al.	0	11.50
		LowMC-GC-PSI	4.2	129.73
		ECC-NR-PSI	4.2	32.71

Optimizing for communication complexity

Frobenius automorphism

- The Frobenius automorphism maps any $y \in \mathbb{F}_{t^d}$ to $\mathrm{Frob}(y, r) = y^{t^r}$.
- This operation introduces much less noise than homomorphic multiplication.
- We can get depth $\mathcal{O}(\log \log D)$ sending only $\mathcal{O}(1)$ pre-computed powers instead of $\mathcal{O}(\log D)$.

Optimizing for communication complexity

Frobenius automorphism

Example

Take a plaintext modulus $t = 2$; the Frobenius operation can compute $[\![x]\!] \mapsto [\![x^{2^i}]\!]$.
Suppose the sender has 255 values in its set.
To use Paterson-Stockmeyer, the sender needs:

- **Low powers** $[\![y]\!]^2, [\![y]\!]^3, \ldots, [\![y]\!]^{15}$
- **High powers:** $[\![y]\!]^{16}, [\![y]\!]^{32}, [\![y]\!]^{48}, \ldots, [\![y]\!]^{240}$

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The receiver sends only $[\![y]\!]$. The sender calculates:

- $[\![y]\!], [\![y^2]\!], [\![y^4]\!], [\![y^8]\!]$ with depth 0.

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- $[\![y^3]\!] = [\![y]\!] \cdot [\![y^2]\!]$, $[\![y^5]\!] = [\![y]\!] \cdot [\![y^4]\!]$, $[\![y^7]\!] = [\![y]\!] \cdot [\![y^2]\!] \cdot [\![y^4]\!]$, $[\![y^9]\!] = [\![y]\!] \cdot [\![y^8]\!]$,

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- $[\![y^{11}]\!] = [\![y]\!] \cdot [\![y^2]\!] \cdot [\![y^8]\!]$, $[\![y^{13}]\!] = [\![y]\!] \cdot [\![y^4]\!] \cdot [\![y^8]\!]$, $[\![y^{15}]\!] = [\![y]\!] \cdot [\![y^2]\!] \cdot [\![y^4]\!] \cdot [\![y^8]\!]$

Optimizing for communication complexity

Results

$\lvert Y \rvert$	Online communication (MB)			
	$\lvert X \rvert = 2^{20}$	2^{22}	2^{24}	2^{26}
1245	2.09	2.28	2.28	2.28
1024 (Chen et al.)	6.45	-	9.02	-
558	1.27	1.27	1.27	1.36
512 (Chen et al.)	5.01	-	10.64	-
341	1.10	1.32	1.32	1.32
256 (Chen et al.)	4.73	-	13.58	-
210	0.72	0.76	0.76	0.76
128 (Chen et al.)	4.69	-	18.32	-
126	0.63	0.63	0.66	-

Optimizing for communication complexity

Results

$\lvert X \rvert$	$\lvert Y \rvert$	**Offline (s)** $T = 24$	**Online (s)** $T = 24$
2^{26}	1245	296	889
	210	1450	1640
2^{24}	1245	64.7	338
	210	305	354
2^{22}	1245	14.1	140
	210	65.2	105
2^{20}	1245	2.88	43.4
	210	14.0	38.7

Conclusion

When intersecting 2^{28} and 2048 item sets:

- Reduced computation by 71%, communication by 63%.

When intersecting 2^{24} and 4096 item sets:

- Reduced computation by 27%, communication by 63%.

PSI with **nearly constant communication** in the larger set size.

➤ Optimizations also apply in the **labeled mode**.

Implementation available at:
`https://github.com/microsoft/APSI/`